

Laws in CSP

Joakim Ahnfelt-Rønne

This is a distillation of some of the laws from Communicating Sequential Processes by Sir Charles Antony Richard Hoare, the inventor of CSP, Hoare logic, Quicksort... and null. The book is freely available online at usingcsp.com.

$P \sqcap P = P$	3.2.1.L1
$P \sqcap Q = Q \sqcap P$	3.2.1.L2
$P \sqcap (Q \sqcap R) = (P \sqcap Q) \sqcap R$	3.2.1.L3
$x \rightarrow (P \sqcap Q) = (x \rightarrow P) \sqcap (x \rightarrow Q)$	3.2.1.L4
$x : B \rightarrow (P(x) \sqcap Q(x)) = (x : B \rightarrow P(x)) \sqcap (x : B \rightarrow Q(x))$	3.2.1.L5
$P \parallel (Q \sqcap R) = (P \parallel Q) \sqcap (P \parallel R)$	3.2.1.L6
$(P \parallel Q) \sqcap R = (P \parallel R) \sqcap (Q \parallel R)$	3.2.1.L7
$f(P \sqcap Q) = f(P) \sqcap f(Q)$	3.2.1.L8

$P \square P = P$	3.3.1.L1
$P \square Q = Q \square P$	3.3.1.L2
$P \square (Q \square R) = (P \square Q) \square R$	3.3.1.L3
$P \square STOP = P$	3.3.1.L4
$(x : A \rightarrow P(x)) \square (y : B \rightarrow Q(y)) =$ $(z : (A \cup B) \rightarrow$ if $z \in (A - B)$ then $P(z)$ else if $z \in (B - A)$ then $Q(z)$ else if $z \in (A \cap B)$ then $(P(z) \sqcap Q(z))))$	3.3.1.L5
$P \square (Q \sqcap R) = (P \square Q) \sqcap (P \square R)$	3.3.1.L6
$P \sqcap (Q \square R) = (P \sqcap Q) \square (P \sqcap R)$	3.3.1.L7

$\text{refusals}(STOP_A) = \text{all subsets of } A \text{ (including } A \text{ itself)}$	3.4.1.L1
$\text{refusals}(c \rightarrow P) = \{X \mid X \subseteq (\alpha P - \{c\})\}$	3.4.1.L2
$\text{refusals}(x : B \rightarrow P(x)) = \{X \mid X \subseteq (\alpha P - B)\}$	3.4.1.L3
$\text{refusals}(P \sqcap Q) = \text{refusals}(P) \cup \text{refusals}(Q)$	3.4.1.L4
$\text{refusals}(P \square Q) = \text{refusals}(P) \cap \text{refusals}(Q)$	3.4.1.L5
$\text{refusals}(P \parallel Q) = \{X \cup Y \mid X \in \text{refusals}(P) \wedge Y \in \text{refusals}(Q)\}$	3.4.1.L6
$\text{refusals}(f(P)) = \{f(X) \mid X \in \text{refusals}(P)\}$	3.4.1.L7
$X \in \text{refusals}(P) \Rightarrow X \subseteq \alpha P$	3.4.1.L8
$\{\} \in \text{refusals}(P)$	3.4.1.L9
$(X \cup Y) \in \text{refusals}(P) \Rightarrow X \in \text{refusals}(P)$	3.4.1.L10
$X \in \text{refusals}(P) \Rightarrow (X \cup \{x\}) \in \text{refusals}(P) \vee \langle x \rangle \in \text{traces}(P)$	3.4.1.L11

$P \setminus \{\} = P$	3.5.1.L1
$(P \setminus B) \setminus C = P \setminus (B \cup C)$	3.5.1.L2
$(P \sqcap Q) \setminus C = (P \setminus C) \sqcap (Q \setminus C)$	3.5.1.L3
$STOP_A \setminus C = STOP_{A-C}$	3.5.1.L4
$(x \rightarrow P) \setminus C =$ if $x \notin C$ then $x \rightarrow (P \setminus C)$ else if $x \in C$ then $P \setminus C$	3.5.1.L5
if $\alpha P \sqcap \alpha Q \sqcap \alpha C = \{\}$ then $(P \parallel Q) \setminus C = P \setminus C \parallel Q \setminus C$	3.5.1.L6
$f(P \setminus C) = f(P) \setminus f(C)$	3.5.1.L7
if $B \cap C = \{\}$ then $(x : B \rightarrow P(x)) \setminus C = x : B \rightarrow (P(x) \setminus C)$	3.5.1.L8
if $B \subseteq C$ and B is finite and not empty then $(x : B \rightarrow P(x)) \setminus C = \prod_{x \in B} (P(x) \setminus C)$	3.5.1.L9
if $C \cap B$ is finite and not empty then $(x : B \rightarrow P(x)) \setminus C =$ $Q \sqcap (Q \sqcap (x : (B - C) \rightarrow P(x)))$ where $Q = \prod_{x \in B \cap C} P(x) \setminus C$	3.5.1.L10
$\text{traces}(P_{+B}) = \text{traces}(P)$	3.5.1.L11
$P_{+B} \setminus B = P$	3.5.1.L12

$(c?v \rightarrow P) \parallel (c!v \rightarrow Q(x)) = c?v \rightarrow (P \parallel Q(v))$	4.3.1.L1
$((c?v \rightarrow P) \parallel (c!v \rightarrow Q(x))) \setminus C = (P \parallel Q(v)) \setminus C$ where $C = \{c.v \mid v \in \alpha c\}$	4.3.1.L2